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SEMI-ACTIVE TUNED MASS DAMPERS FOR SEISMIC PROTECTION OF CIVIL STRUCTURES

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SUMMARY

A semi-active tuned mass damper (TMD) which is capable of adjusting initial displacement and damping is applied to seismic protection of structures. Control algorithms are derived in closed forms using perturbation analysis on modal properties of the single degree of freedom (SDOF) structure/TMD system. Because perturbation solutions are given in a relatively simple form, the control algorithms are developed in a physically intuitive manner. In the first half of the paper, the control algorithms are introduced using numerical simulations of the impulse response. Then the algorithms are applied to seismic protection of civil structures using the strategy of multiple releasing and capturing of TMDs. The control strategy for seismic control is summarized in a flow chart. Numerical studies with the El Centro earthquake record show that the proposed semi-active method has higher performance than conventional passive TMDs.

KEY WORDS: perturbation solutions; seismic protection; semi-active control; structural control; tuned mass dampers

INTRODUCTION

Tuned mass dampers (TMD) are widely used to control vibrations in civil structures. Although TMDs are effective in reducing vibrations caused by stationary excitation forces, their performance to suppress seismic response is relatively limited.[1–3] This inefficiency is due to the fact that TMDs usually need some time interval before it becomes fully effective because they are initially at rest, while the strongest seismic ground motion is often observed at the earlier stage of an earthquake. Although applying active TMDs can solve this problem,[4,5] active systems require large control force and have problems concerning reliability.

In order to reduce required control force and improve reliability, semi-active TMDs have also attracted attention from many researchers. Several implementations of semi-active TMDs have been proposed: (i) TMDs with initial TMD displacements;[6] (ii) TMDs with variable damping components;[7] and, (iii) TMDs with pulse generators.[8] All of these control devices require a limited amount of control force and show better performance than their passive counterparts. Another advantage of these devices is that they can work as passive devices in case of power failure. However, the regular linear control theory cannot be applied to these semi-active systems, because they have non-linear characteristics. Hence, control laws are usually constructed through (i) linearization; (ii) trial-and-error procedure by numerical simulations and experiments; or (iii) physical intuition.

In this paper, the semi-active strategy which utilizes both of (i) the initial TMD displacement and (ii) the variable damping is applied to seismic protection. Control strategy is developed based upon the optimal semi-active control laws for transient vibration suppression.[9] The control law is derived from perturbation solutions of the modal properties of the single degree of freedom (SDOF) structure/TMD system.[10,11] The derived initial TMD displacement and TMD damping ratio has been shown to maximize the exponential decay rate of the structural response.[9] The concept of the control law will be briefly reviewed with the examples of impulse response of the SDOF structure/TMD system. Because all the results are developed in a simple analytical form, the control law can be explained in a physically intuitive manner. Although the control law is developed for SDOF structures, it can easily be applied to multiple degrees of freedom (MDOF) or continuous structures when natural frequencies of the structure are widely spaced.[12,13]

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To demonstrate the effectiveness of the proposed method, numerical simulations of an SDOF structure subject to El Centro earthquake are performed. In the simulations, it is assumed that (i) TMD damping can be adjusted upon releasing from the initial position; (ii) TMD can be captured when its velocity is zero; (iii) TMD can be readjusted to the specified position with a finite speed; (iv) TMD can be released again after readjustment. This assumption follows experimentation[6] except that damping adjustment is included. By numerical simulations using the El Centro earthquake record, the semi-active TMDs will be shown to be more effective in reducing the maximum response of the structures than conventional passive TMDs.

PERTURBATION SOLUTIONS OF VIBRATION MODES

In this section, perturbation solutions of modes for the SDOF structure/TMD system are briefly reviewed. Detailed derivation can be found in References 10 and 11.

Consider the SDOF structure/TMD system shown in Figure 1. Masses and frequencies are denoted by m_s and ω_s for the structure, and m_t and ω_t for the TMD. The optimal natural frequency of the TMD is given approximately by $\omega_t = \omega_s/(1 + \mu)$ when the mass ratio $\mu = m_t/m_s$ is small.[14,15] Hence, this value is used as the design natural frequency of the TMD in the following analysis. The TMD has the damping ratio ζ. Damping of the structure is neglected because TMD design parameters are only slightly affected by structural damping when structural damping is small.[10] Besides, structures which need vibration control usually have extremely low damping. The TMD displacement y is taken relative to the structural displacement x.

Perturbation analysis is based upon the assumption that (i) the TMD frequency is tuned or nearly tuned to the structural frequency so that both natural frequencies can be approximated by their average, $\omega_s \approx \omega_t \approx \omega_a \equiv (\omega_s + \omega_t)/2$; (ii) the TMD mass is small compared to the structural mass; (iii) the TMD damping ratio is small relative to unity. By applying the perturbation analysis using this assumption, the two modal frequencies and damping ratios of the system can be expressed in terms of ζ as,

$$\omega_{1,2} = \omega_a(1 \pm \mathrm{Im}\,\gamma/2); \qquad \zeta_{1,2} = (\zeta \pm \mathrm{Re}\,\gamma)/2 \tag{1}$$

where $\gamma = \sqrt{\zeta^2 - \mu}$. The corresponding mode shapes are,

$$\begin{pmatrix} \phi_x \\ \phi_y \end{pmatrix}_{1,2} = \begin{pmatrix} 1 \\ -\,\mathrm{i}(\zeta \pm \gamma)/\mu \end{pmatrix} \tag{2}$$

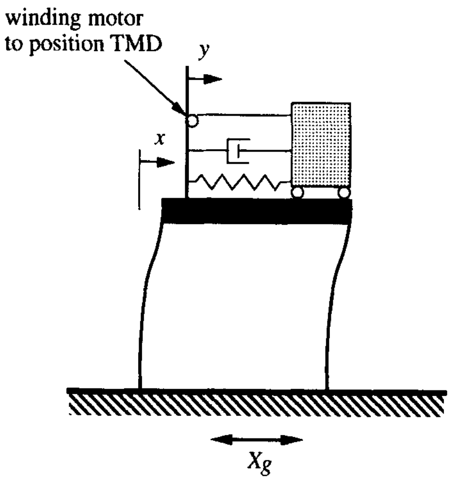

Figure 1. SDOF structure/semi-active TMD system

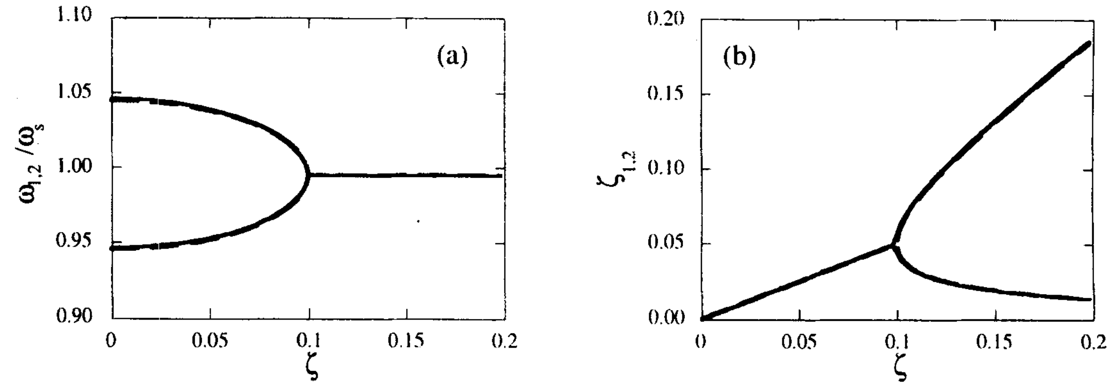

Figure 2. Modal properties of the SDOF structure/TMD system ($\mu = 0{\cdot}01$). (a) modal frequencies, (b) modal damping ratios. Solid line corresponds to numerical solutions and the dashed line to perturbation solutions

The modal frequencies and damping ratios are plotted in Figure 2 for $\mu = 0{\cdot}01$. Both modal damping ratios are monotonically increasing until $\zeta = \sqrt{\mu}$. When $\zeta > \sqrt{\mu}$, ζ_1 still increases, although ζ_2 approaches zero. The value of ζ to maximize both modal damping ratios is $\zeta = \sqrt{\mu}$, which gives the modal damping ratios of $\zeta_1 = \zeta_2 = \sqrt{\mu}/2$. Increasing ζ beyond $\sqrt{\mu}$ makes TMDs less effective, because of the effect of the lowly damped second mode. Therefore, $\zeta = \sqrt{\mu}$ is employed as the passive optimal damping ratio of the TMD in this paper.

SEMI-ACTIVE CONTROL LAW

In this section, control of impulse response of the SDOF structure/TMD system is studied to form the basis of the seismic problems. Initial conditions are specified by $x(0) = 0$, $y(0) = y_0$, $\dot{x}(0) = \dot{x}_0$, and $\dot{y}(0) = \dot{y}_0$, where y_0 is the possible initial TMD displacement. As noted in the preceding section, increasing the TMD damping ratio ζ beyond $\sqrt{\mu}$, is not effective due to the lowly damped second mode.The basic strategy is to choose appropriate y_0 and ζ to eliminate this undesirable second mode and to excite only the highly damped first mode.

To excite the first mode without introducing the second mode, the structural response x and the TMD response y should be in the shape of the first mode. In other words, they should satisfy the relationship

$$y/x = \mathrm{Re}[-\mathrm{i}(\zeta + \gamma)/\mu] \tag{3}$$

by the mode shape given in equation (2). For a short duration of time after $t = 0$, the responses of the structure and the TMD can be approximated by,

$$x(t) = \mathrm{Re}[\dot{x}_0 \mathrm{e}^{\mathrm{i}\omega_a t}/(\mathrm{i}\omega_a)], \qquad \dot{x}(t) = \mathrm{Re}[\dot{x}_0 \mathrm{e}^{\mathrm{i}\omega_a t}], \tag{4a,b}$$

$$y(t) = \mathrm{Re}[y_0 \mathrm{e}^{\mathrm{i}\omega_a t}], \qquad \dot{y}(t) = \mathrm{Re}[(\mathrm{i}\omega_a)\dot{y}_0 \mathrm{e}^{\mathrm{i}\omega_a t}]. \tag{5a, b}$$

By the relation of equations (4) and (5), y/x can be expressed as,

$$y/x = \mathrm{Re}[\mathrm{i}\omega_a y_0/\dot{x}_0]. \tag{6}$$

Equating the right-hand sides of equations (3) and (6) yields,

$$y_0 = -(\zeta + \gamma)\dot{x}_0/(\mu\omega_a) \tag{7}$$

Equation (7) gives the optimal initial TMD displacement when TMD damping ratio ζ is fixed. For larger values of ζ where $\zeta \gg \sqrt{\mu}$, equation (7) can be further simplified to,

$$y_0 \approx -2\zeta\dot{x}_0/(\mu\omega_a). \tag{8}$$

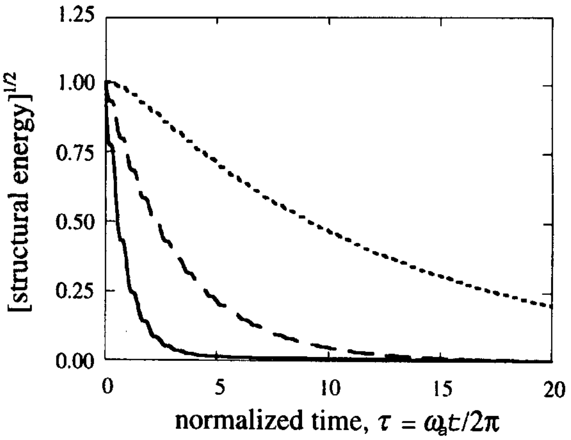

Figure 3. Impulse response of the structure with and without TMD. ———: with semi-active TMD ($\mu = 0{\cdot}01$, $\zeta = 0{\cdot}2$), - - - - - -: with passive TMD ($\mu = 0{\cdot}01$, $\zeta = 0{\cdot}2$), — — —: with optimal passive TMD ($\mu = 0{\cdot}01$, $\zeta = 0{\cdot}1$)

If the initial TMD displacement is fixed and if the TMD damping is variable, the desirable damping ratio is given by rewriting equation (8) as,

$$\zeta \approx -\mu\omega_a y_0/(2\dot{x}_0) \tag{9}$$

Since ζ must be positive to stabilize the system, the directions of the initial TMD displacement and the initial structural velocity must be opposite. The initial displacement of equations (7) or (8), and the damping ratio of equation (9) are employed as the semi-active control law.

The impulse response of the SDOF structure/TMD system with $\mu = 0{\cdot}01$ and $\zeta = 0{\cdot}2$ is given in Figure 3. Here, the TMD damping ratio ζ is fixed, and the control law of equation (7) is employed. By equation (7), the initial displacement of the TMD is set to $y_0 = -37{\cdot}3\dot{x}_0/(\omega_a)$. Time is normalized by the average natural period $2\pi/\omega_a$, and the structural response is expressed in terms of the square root of vibration energy divided by the initial energy, which shows the envelope process of the response. The application of the initial TMD displacement is shown to improve the performance of the TMD considerably.

APPLICATION TO SEISMIC PROTECTION

Here, the results of the preceding section are applied to seismic problems. A TMD is released and captured several times during an earthquake, in order to make the semi-active TMD effective throughout the earthquake. The initial TMD displacement is fixed and the TMD damping ratio is adjusted to each releasing of TMD, so the control law of equation (9) is employed. The flow chart for the seismic control strategy is given in Figure 4. The TMD is released from the position of $y_0 (> 0)$, when the structural velocity exceeds certain threshold value as $\dot{x} < \dot{x}_{\mathrm{threshold}}$ (< 0) at the zero crossing of structural displacement x. Note that $\dot{x}_{\mathrm{threshold}}$ and y_0 should be set in opposite directions to make the damping ratio given by equation (9) positive. About half a cycle after the releasing, the TMD velocity becomes approximately zero. At this moment, the TMD is captured without introducing any excessive reaction force. Then, TMD is readjusted to the original position y_0 with a constant velocity V_a. In this way, TMD is ready for another release. Impulse $m_t V_a$ is experienced by the structure as the reaction at the beginning and end of the positioning of the TMD. This modelling is found to give good approximation when the acceleration time of the mass is much shorter than the natural period of the structure.

Responses of an SDOF structure subject to the El Centro earthquake with the peak ground acceleration of $0{\cdot}34\ \mathrm{m/s^2}$ are studied numerically to demonstrate the characteristics of the proposed method. The

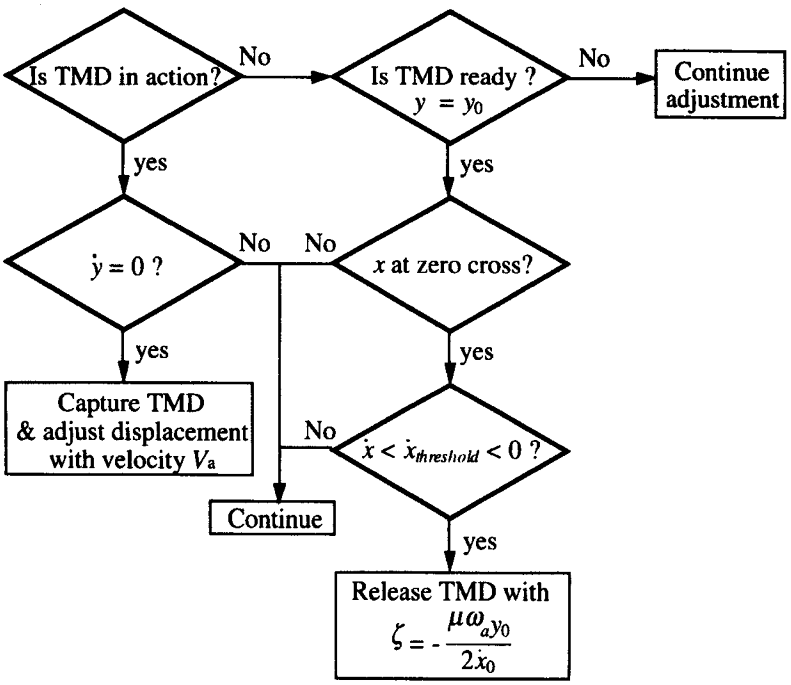

Figure 4. Flow chart for seismic structural control with semi-active TMD

structure has the natural period of 1 s and damping ratio of 1 per cent. The TMD mass ratio μ is 0·01. The value of y_0 is chosen as 2 m based on the room available for TMDs in ordinary high rise buildings. The positioning speed V_a is taken as 1 m/s to reflect the performance of available motors. However, the size and the cost of the motors to achieve this specification can be enormous for TMDs with large masses, which should be noted in application to real civil structures. The threshold structural velocity $\dot{x}_{\mathrm{threshold}}$ is set to $-0{\cdot}02\omega_s$ m/s to approximate the structural displacement of $x \approx 0{\cdot}02$ m. If the structural displacement is smaller than this value, most buildings are expected to be undamaged and the response reduction by control will not be necessary.

Figure 5(a) shows the response without a TMD and with a passive TMD. As commonly observed in seismic problems, the TMD reduces the maximum structural response only slightly. Figure 5(b) shows the response with the semi-active TMD with TMD displacement $y_0 = 2$ m, the positioning speed $V_a = 1$ m/s, and $\dot{x}_{\mathrm{threshold}} = 0{\cdot}02\omega_s$ m/s. The semi-active TMD is observed to reduce the maximum response more than the conventional passive TMD. The corresponding TMD position is shown in Figure 5(c). The TMD is released five times during the first 20 s of the El Centro earthquake. The straight line portion with slope 1 m/s corresponds to the positioning process of the TMD. The TMD cannot be released during this time interval.

To see the effect of the TMD displacement y_0, the maximum structural displacement is plotted for various values of y_0 in Figure 6. Although larger values of y_0 are unrealistic, they are also plotted to show the tendency clearly. Generally, the maximum response decreases with larger y_0. However, an extremely large y_0 does not necessarily give more reduction in response. This is because V_a is fixed and it needs more time to readjust TMD for a larger value of y_0.

In the preceding study, initial TMD displacement y_0 is fixed, so the semi-active TMD can be released only in a fixed direction. Natural extension of this technology is to apply two semi-active TMDs with initial displacements of $\pm y_0$, so that they can be released in both directions. The results for these two-way semi-active TMDs are also shown in Figure 6. Each TMD has a mass ratio of 0·005 to keep the same total mass ratio as the one-way TMD system. The two-way system gives higher reduction for the range of small y_0, but mostly gives almost the same performance as the one-way system. Although the two-way system can

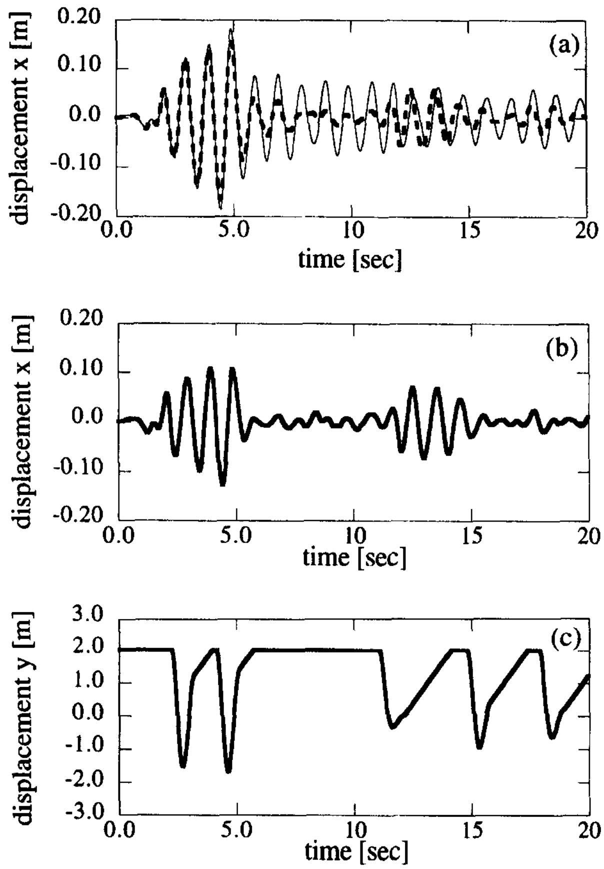

Figure 5. Structural response subject to El Centro earthquake with and without TMD. (a) ———: without TMD, – – – – – –: with optimal passive TMD ($\mu = 0{\cdot}01$, $\zeta = 0{\cdot}1$); (b) with semi-active TMD ($\mu = 0{\cdot}01$, $y_0 = 2$ m, $V_a = 1$ m/s); (c) motion of semi-active TMD ($\mu = 0{\cdot}01$, $y_0 = 2$ m, $V_a = 1$ m/s)

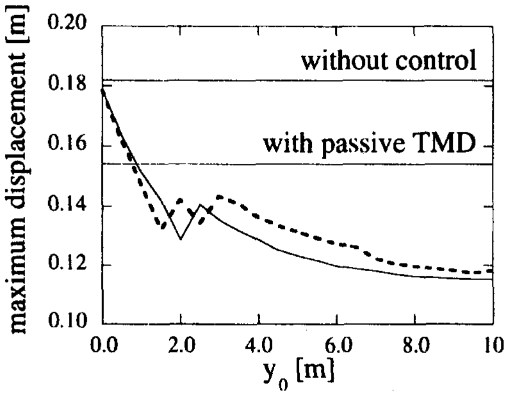

Figure 6. Effect of y_0 on maximal structural response subject to El Centro earthquake. ———: with one-way semi-active TMD ($\mu = 0{\cdot}01$, $V_a = 1$ m/s); – – – – – –: with two-way semi-active TMDs ($\mu = 0{\cdot}005 \times 2$, $V_a = 1$ m/s)

have more opportunities to release TMDs, each TMD has half the mass ratio of the one-way TMD and it has less vibration suppression effect. This fact is the reason that the two-way system does not improve the results significantly.

CONCLUSIONS

The performance of semi-active tuned mass dampers with initial TMD displacement and variable damping subject to earthquake excitation is studied. The algorithms are developed in a simple closed form using the perturbation solutions of vibration modes. The control algorithm is extended to seismic structural control by the strategy shown in the chart of Figure 4. Performance of the proposed method is demonstrated by numerical simulations using the model of the SDOF structure/TMD system subject to impulse loading and earthquake loading. In both loading cases, semi-active TMDs give higher reduction of structural response than conventional passive TMDs. Although increasing the initial TMD displacement generally reduces the structural response, an extremely large TMD displacement does not necessarily give better performance. The reason is that positioning of the TMD takes more time with a larger TMD displacement when the positioning speed is fixed. Two semi-active TMDs whose directions of initial displacements are opposite to each other are also studied. This two-way system has almost the same performance as a one-way system with the same total mass ratio.

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